COST OPTIMIZATION OF VERTICAL NATURAL-CIRCULATION STEAM GENERATORS

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A mathematical model for the cost optimization of U-tube vertical natural-circulation steam generators, used in pressurized-water nuclear power plants, is developed. The total annual cost function is expressed as a function of the heat exchange area and the pumping power. Parametric studies indicate that the global minimum cost is on the line of the lowest constant inside diameter and the lowest constant inside surface roughness. This mathematical formulation is useful for incorporation in the cost optimization of the entire nuclear power plant.

1. Introduction

Optimization of heat exchanger design is of interest in many engineering applications. A poor choice of design parameters, such as the number of tubes and the tube diameter, can adversely affect the cost of the heat exchanger and thereby reduce the economic effectiveness of the heat transfer equipment [1].

A computerized design of the minimum cost of a cross-flow gas cooler was undertaken by Mott et al. [2]. The computer program developed provides a method for rapidly evaluating the economic potential of available heat transfer matrices for a particular heat exchanger application. A similar study was conducted by Dehne [3] for the economical design of air-cooled heat exchangers. He employs an optimization program on many types of heat exchangers to arrive at the most economical solution for a given heat transfer problem.

LaHaye et al. [4] showed a new method of presenting the heat transfer data for the prediction of heating surface performance and heat exchanger optimization. Their analysis leads to a dimensionless performance plot between a ‘heat transfer performance factor’ and a ‘pumping power factor’ with a non-dimensional ‘flow length between major boundary layer disturbances’ as a varying parameter. This plot permits the rapid assessment and comparison of heat transfer geometries for a given application and is valuable in optimizing a design accounting for space limitation, economic restraints and system considerations such as pumping power and effectiveness trade-offs.

Computer optimization of dry cooling tower heat exchangers has been undertaken by many authors. Andeen and Glicksman [5] developed a digital computer program for the optimization of the entire power plant considering all interactions between boiler, turbine and heat exchanger. They found the optimum point where the incremental increase in cost of power generation, as a result of the application of the dry cooling towers, is a minimum. Armstrong and Schermerhorn [6] studied the economic effects of dry cooling towers when applied to an unfired combined-cycle plant. They showed that for this particular case, the dry towers offer a minor cost penalty as compared to other alternative cooling methods.

Optimization of heat transfer designs by geometric programming has been performed by Oswald and Kochenberger [7]. Applications are shown for the selection of heat transfer media considering power requirements, heat exchanger cost, pipe diameter, velocity, temperature and physical properties.

The purpose of this study is to examine the cost optimization of the U-tube natural-circulation steam generators used in pressurized-water nuclear power stations. In particular, the objective is to show the effects of the number of tubes, the inside diameter and the roughness on the cost of the steam generating units. The mathematical formulation developed in this study is useful for incorporation in the cost optimization of the entire nuclear power plant when considering the interaction between the reactor vessel, steam generators, main coolant pumps and the pressurizer.
2. Mathematical formulation

Figure 1 is a schematic diagram of a vertical U-tube natural-circulation steam generator. The primary coolant flows through the inside of many hundreds of small stainless-steel tubes in the heat exchanger section of the steam generator. These heat exchanger tubes are surrounded by the water of the secondary system, which is heated by the primary coolant. Wet steam is formed which flows upward through the riser and enters the steam separator portion of the steam generator. Here the moisture is removed and returned (together with the cold water feed) to the heat exchanger section through the downcomer. The dry and saturated steam leaves the top of the steam dryer and goes to the steam turbine. The mathematical formulation is based on the following assumptions:

1. The primary and the secondary coolants are at subcooled and two-phase flow conditions, respectively. The secondary coolant temperature is treated as a constant.

2. The steam generating unit is considered to be continuously working at steady-state operating conditions.

The cost of a U-tube natural-circulation steam generator $C$ can be expressed as a function of the heat exchange area $A$ and the pumping power per unit area $E$, or

$$C = A C_a R + A E \theta C_p ,$$

where the heat exchange area $A$ and the pumping power per unit area $E$ are calculated on the basis of the steam generator thermal and hydraulic design considerations, respectively. The capital recovery factor $R$, cost per unit area $C_a$, and cost per unit energy $C_p$ are determined on the basis of economics, equipment and power costs, respectively. The pump running time $\theta$ is established on the basis of the operating time, assumed to be continuous. The calculation of these parameters is discussed in the following sections.

2.1. Heat exchange area

Applying an energy balance on a small U-tube element along the primary coolant flow gives

$$Q = dq/dA = W_p c_p (dT_p/dA) ,$$

$$Q = U(T_p - T_s) .$$

The inverse of the overall heat transfer coefficient $U$ (from the primary to the secondary fluid) is given as the sum of the primary film, tube wall, fouling factor and secondary film resistances

$$1/U = R_p + R_w + R_f + R_s = R_c + R_s .$$

It should be noted that since the boiling film resistance $R_s$ is dependent on the heat flux, the integration of eqs. (2a) and (2b) is not straightforward.

The primary film resistance is given by

$$R_p = D_o/D_p h_p ,$$

where the primary film convective heat transfer coeff-
The tube wall resistance is calculated from

\[ R_w = \frac{D_o \ln (D_o/D_i)}{Z_k w}. \]  

(6)

The fouling factor is considered constant. The secondary film resistance is calculated by combining the Rohsenow's boiling heat transfer correlation \[9\]

\[ \frac{c_f (T_w - T_s)}{h_{fg}} = c_{ws} \left[ \left( \frac{Q}{\mu_f h_{fg}} \left( \frac{g c}{\rho_f - \rho_g} \right)^{1/2} \right) \right]^{2/3} \]  

(7)

or simply

\[ T_w - T_s = F(P_s) Q^{1/3} \]  

(8)

with the secondary heat flux relation

\[ Q = (T_w - T_s)/R_s \]  

(9)

to obtain

\[ R_s = F(P_s) Q^{-2/3}. \]  

(10)

Combining eqs. (2b), (3) and (10) yields

\[ T_p - T_s = R_c Q + F(P_s) Q^{1/3}. \]  

(11)

Integrating eq. (2a) and noting that \(T_s\) is constant gives

\[ A = \int \frac{1}{W_p c_p} \left[ d(T_p - T_s)/Q \right]. \]  

(12)

Substituting eq. (11) into (12) and performing the integration, the heat exchange area is obtained in terms of the tube heat fluxes at the inlet and outlet:

\[ A = W_p c_p \left[ \frac{Q_2}{Q_1} \right] \left[ R_c \ln \left( \frac{Q_1}{Q_2} \right) + \frac{1}{2} F(P_s) (Q_2^{2/3} - Q_1^{-2/3}) \right]. \]  

(13)

Employing eq. (11), the tube heat fluxes at the inlet and outlet \(Q_1\) and \(Q_2\) are found from eqs. (14) and (15) by the Newton–Raphson iterative method:

\[ T_{p1} - T_s = R_c Q_1 + F(P_s) Q_1^{1/3}, \]  

(14)

\[ T_{p2} - T_s = R_c Q_2 + F(P_s) Q_2^{1/3}. \]  

(15)

2.2. Pumping power per unit area

The pumping power per unit area is calculated in terms of the frictional power losses expended per unit surface to pump the primary coolant through the heat exchanger tubes

\[ E = (A_i/A) (f \rho_p V_p^3) \]  

(16)

The primary coolant velocity \(V_p\) is related to other parameters by

\[ W_p = \frac{\pi D_i^2 N \rho_p V_p}{4}. \]  

(17)

The friction factors for smooth and rough conditions are calculated from the following correlations, respectively, which agree with the well-known Moody's chart:

\[ f = 0.184 (W_p/D_i/A_p \mu_p)^{-0.2} \]  

(18)

and

\[ f = 0.0055 \left[ 1 + \left( 2 \times 10^4 \frac{e}{D_i} + \frac{10^6 A_p \mu_p}{W_p D_i} \right)^{0.333} \right]. \]  

(19)

The tube inside and outside diameters are related by the ASME Power Boiler Code formula for wall thickness for high-pressure, high-temperature piping \[10\]

\[ t = (D_o - D_i)/2 = P_p D_o/(2S + 2y P_p). \]  

(20)

2.3. Capital recovery factor

The capital recovery factor \(R\), the sum of the return (on the invested capital) and sinking fund depreciation components, is a function of the interest rate \(i\) and the steam generator life \(n\) \[11\], i.e.

\[ R = i(1+i)^n/[1+(1+i)^n - 1]. \]  

(21)
Table 1.
Steam generator specification.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>British units</th>
<th>SI units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat transfer load</td>
<td>(4.18 \times 10^9) Btu/hr</td>
<td>1225 MW</td>
</tr>
<tr>
<td>Primary flow, (W_p)</td>
<td>(62.25 \times 10^6) lb/hr</td>
<td>7.843 kg/sec</td>
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<tr>
<td>Coolant inlet temperature, (T_{p1})</td>
<td>598.5°F</td>
<td>314.7°C</td>
</tr>
<tr>
<td>Coolant outlet temperature, (T_{p2})</td>
<td>547.8°F</td>
<td>286.6°C</td>
</tr>
<tr>
<td>Primary pressure, (P_p)</td>
<td>2100 psia</td>
<td>14.48 \times 10^6) Pa</td>
</tr>
<tr>
<td>Secondary pressure, (P_s)</td>
<td>770 psia</td>
<td>5.31 \times 10^6) Pa</td>
</tr>
<tr>
<td>Secondary temperature, (T_s)</td>
<td>513.8°F</td>
<td>267.7°C</td>
</tr>
<tr>
<td>Fouling factor, (R_f)</td>
<td>0.0003 hr-ft^2°F/Btu</td>
<td>5.283 \times 10^6 m^2-C/W</td>
</tr>
<tr>
<td>Range of number of U-tubes, (N)</td>
<td>200–50000</td>
<td>200–50000</td>
</tr>
<tr>
<td>Range of U-tube i.d., (D_i)</td>
<td>0.4–1.5 in</td>
<td>10.16–38.10 mm</td>
</tr>
<tr>
<td>U-tube roughness, (e)</td>
<td>0–0.0001 ft</td>
<td>0–0.03 mm</td>
</tr>
</tbody>
</table>

2.4. Unit costs

The steam generator cost per unit area \(C_a\) is obtained from the so-called 'six-tenths rule', which has given good results in estimating capital equipment costs [7]

\[
C_a/C_{a0} = (A/A_0)^{0.6} \tag{22}
\]

This rule gives the unit cost of a new piece of equipment in terms of the unit cost of another capacity for which cost data are at hand. The cost per unit energy \(C_p\) is considered constant.

3. Presentation of results and conclusions

The mathematical formulation derived in the previous section is applied to the analysis of a typical U-tube vertical steam generator, used in pressurized-water reactor systems, specified in table 1. To facilitate this application, two digital programs were developed using the above formulation:

1. A parametric study program which could vary one or more design parameters at a time and determine the cost in terms of these parameters. This program proved to be useful in gaining insight into the nature of the problem.

2. An optimization study program using an available direct search program for automated optimal design [12].

The mathematical formulation presented herein is sufficiently general to optimize the system with respect to all design variables. However, the optimization procedure is performed with only three design variables — number of U-tubes, their inside diameter and their roughness.

Table 2.
Input parameters used in the analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>British units</th>
<th>SI units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost per unit area of the</td>
<td>60.00 $/ft^2</td>
<td>645.83 $/m^2</td>
</tr>
<tr>
<td>reference steam generator, (C_{a0})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat exchange area of the</td>
<td>50 000 ft^2</td>
<td>4645.15 m^2</td>
</tr>
<tr>
<td>reference steam generator, (A_0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost per unit energy, (C_p)</td>
<td>0.012 $/kWhr</td>
<td>3.33 \times 10^{-6} $/J</td>
</tr>
<tr>
<td>Interest rate, (i)</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>Steam generator life, (n)</td>
<td>40 yr</td>
<td>40 yr</td>
</tr>
<tr>
<td>Allowable stress in U-tubes, (S)</td>
<td>23 300 psia</td>
<td>160.6 \times 10^6) Pa</td>
</tr>
<tr>
<td>Coefficient (y) of the ASME Boiler Code</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Steam generator operating time, (\theta)</td>
<td>8760 hr</td>
<td>31.536 \times 10^6) sec</td>
</tr>
</tbody>
</table>
The effects of the number of tubes and their inside diameter and roughness on the following design variables were examined:

1. heat exchange area and its contribution to the total cost (figs. 2 and 3);
2. reactor coolant pumping power and its contribution to the total cost (figs. 4 and 5);
3. total cost (fig. 6); and
4. the effect of tube roughness (figs. 7 and 8).

Figure 2 shows the variation of the heat exchanger area versus the number of tubes, with inside diameter considered as a parameter. The heat exchange area required to transfer a constant thermal load decreases with a decrease in the number of tubes and their in-
side diameter, as expected. The reduction of the tubes' inside diameter, with a constant reactor coolant flow rate, increases the coolant velocity with a subsequent increase in heat transfer capability leading to a reduction of heat exchange area. Furthermore, the reduction in the number of tubes increases the reactor coolant velocity, with an attendant reduction in the heat exchange area. The contribution of the heat exchange area to the amortized capital cost (the first term on the right-hand side of eq. (1)) is shown in fig. 3. These curves exhibit a pattern similar to the heat exchange area curves.
Figure 4 shows the variation of the coolant pumping power per unit area versus the number of tubes, with inside diameter considered as a parameter. The pumping power required to transfer a constant thermal load increases with a decrease in the number of tubes and the tube inside diameter, as expected. The reduction in the number of tubes and their inside diameter, with a constant reactor coolant flow rate, increases the reactor coolant velocity, leading to an increase in the pumping power required. The contribution of the coolant pumping power to the operating cost (the second term in the right-hand side of eq. (1)) is shown in fig. 5. These
Fig. 5. Annual operating cost of the steam generator versus number of tubes with inside diameter as a parameter.

curves exhibit a pattern similar to the pumping power curves.

Finally, the total annual cost of the steam generator is presented in fig. 6. These curves show the variation of the cost versus number of tubes and their inside diameter. The lines of constant diameter have local minima which form a skewed curve oriented in the ‘northwest’ direction, tilted upward for smaller numbers of tubes. The global minimum cost is located on the line of lowest constant inside diameter. This parametric study is confirmed by the optimization program using the direct search technique. It is apparent that increasing the diameter increases the cost. For this reason, characteristics above 1.5 in. i.d. are not shown.
Although the minimum cost decreases with decreasing inside diameter, technical considerations such as scale formation and maintenance requirements prohibit the use of small diameter tubes.

A parametric study was conducted to determine the effect of pipe roughness on the total annual cost. The loci of the minimum cost points for the constant diameter contours are shown in fig. 7 at two different tube roughnesses $e = 0.0001$ and 0.000005 ft. Fig. 8 shows plots of the local minima at two different tube roughnesses: (1) fully smooth tube surface and (2) tube roughness equal to 0.000005 ft. These curves show that for each tube diameter, the number of tubes and the total annual cost required to transfer the thermal load are smaller for the smooth pipe. This conclusion agrees with the accepted engineering practice that smooth tubing is the most efficient means for heat transmission [13].

**Nomenclature**

- $A$ = total heat exchange area (based on tube o.d.) ($m^2$)
- $A_i$ = total heat exchange area (based on tube i.d.) ($m^2$)
- $A_p$ = primary coolant flow cross sectional area ($m^2$)
- $A_0$ = total heat exchange area (based on o.d.) for a steam generator whose cost per unit area $C_{a0}$ is known ($m^2$)
Fig. 7. Loci of the minimum cost with tube roughness as a parameter.

\[
\begin{align*}
C &= \text{total yearly steam generator cost (S/yr)} \\
C_a &= \text{cost of steam generator per unit area (S/m}^2) \\
C_{a0} &= \text{a known reference cost of a steam generator with a total heat exchange area } A_0 \text{ (based on tube o.d.)} \\
C_p &= \text{cost per unit energy (S/ws)} \\
c_p &= \text{primary coolant specific heat (J/(kg K))} \\
c_f &= \text{specific heat of saturated liquid for the secondary coolant (J/(kg K))} \\
c_{ws} &= \text{coefficient depending on wall/fluid combination for the secondary coolant (suggested value 0.013, [9])} \\
D_o &= \text{U-tube o.d. (m)} \\
D_i &= \text{U-tube i.d. (m)} \\
E &= \text{pumping power per unit area (W/m}^2) \\
F(P_s) &= \text{a function of pressure defined by eqs. (7) and (8) } (\text{C}^2/(1^{1/3} m^{2/3})) \\
f &= \text{friction factor for primary coolant flow} \\
g_c &= 32.2 \text{ lbm ft/lbf sec}^2 \text{ (equal to dimensionless unity in SI units)} \\
g &= 32.2 \text{ ft/sec}^2 \text{ (equal to 9.81 m/sec}^2 \text{ in SI units)} \\
h_{fg} &= \text{latent heat of evaporation for the secondary coolant (J/kg)} \\
h_p &= \text{primary coolant film convective heat transfer coefficient (W/(m}^2 K)) \\
i &= \text{interest rate (yr}^{-1}) \\
k_p &= \text{primary coolant thermal conductivity (W/(m K))} \\
k_f &= \text{thermal conductivity of saturated liquid for the secondary coolant (W/(m K))} \\
k_w &= \text{U-tube wall thermal conductivity (W/(m K))} \\
N &= \text{number of U-tubes} \\
n &= \text{steam generator life, yr} \\
P_p &= \text{primary coolant pressure (Pa)} \\
q &= \text{heat transfer between primary and secondary coolants (W)} \\
Q &= \text{heat flux between primary and secondary coolants } Q = dq/dA, (W/m}^2) \\
Q_1 &= \text{heat flux at the U-tube inlet (W/m}^2) \\
Q_2 &= \text{heat flux at the U-tube outlet (W/m}^2) \\
R &= \text{capital recovery factor (yr}^{-1}) \\
R_f &= \text{fouling factor ((m}^2 K)/W) \\
R_p &= \text{primary coolant film resistance ((m}^2 K)/W) \\
R_s &= \text{secondary coolant film resistance ((m}^2 K)/W) \\
R_w &= \text{U-tube wall resistance ((m}^2 K)/W) \\
R_c &= R_f + R_p + R_w \\
S &= \text{allowable stress due to the tube internal pressure at the operating temperature (Pa)} \\
T_p &= \text{primary coolant temperature (T}_{p1} = \text{primary} \\
\end{align*}
\]
coolant at the U-tube inlet, $T_{p1}$ = primary coolant at the U-tube outlet) (K)

$T_s$ = secondary coolant temperature (K)

$t$ = U-tube wall thickness (m)

$U$ = primary to secondary coolant overall heat transfer coefficient (W/(m$^2$K))

$V_p$ = primary coolant velocity (m/sec)

$W_p$ = primary coolant mass flow rate (kg/sec)

$\gamma$ = a coefficient given by the ASME Boiler Code

Greek symbols

$\epsilon$ = absolute roughness for the interior surface of
the U-tubes (m)

\[ \theta = \text{steam generator operating time (sec/yr)} \]
\[ \mu_p = \text{primary coolant viscosity (Pa sec)} \]
\[ \rho_f = \text{density of saturated liquid for the secondary coolant (kg/m}^3) \]
\[ \rho_g = \text{density of saturated vapor for the secondary coolant (kg/m}^3) \]
\[ \sigma = \text{surface tension of liquid–vapor interface for the secondary coolant (N/m)} \]

References