The air surrounding a projectile affects the projectile’s motion in three very different ways: the drag force, the buoyant force, and the added mass. The added mass is an increase in the projectile’s inertia from the motion of the air around it. Here we experimentally measure the added mass of a spherical projectile in air. The results agree well with the calculation of the added mass for an ideal fluid. By accounting for the added mass, high school or undergraduate students can easily measure the acceleration of gravity accurately with a beach ball projectile.

For a falling ball in the air, Newton’s law \( F = ma \) takes the form

\[
W - B - D = [m + m_a]a, \tag{1}
\]

where \( W = mg \) is the weight of the ball in vacuum, \( m \) is the mass of the ball in vacuum, \( g \) is the acceleration of gravity, \( B = m_{air}g \) is the buoyant force, \( m_{air} \) is the mass of the air displaced by the ball, \( D \) is the drag force (a function of the ball’s velocity), \( a \) is the downward acceleration, and \( m_a \) is the added mass. All of these terms should be familiar to physics students, except probably the added mass. It is not discussed in introductory physics textbooks and is usually overlooked\(^{1,2}\) in discussions of projectile motion.

The added mass and the drag force both come from the motion of the air around a projectile. The drag force is similar to a friction force; it results from the dissipation of the projectile’s energy. The added mass is an inertial effect that is present even when there is no dissipation. A projectile in air has a larger effective mass because when one accelerates the projectile, one also has to accelerate the air around it. Such inertial effects are common in physics. An effective mass can be used to account for oscillations with a massive spring.\(^3\) Similarly, electrons in a crystal often move like a free particle with an effective mass that is different from their mass in a vacuum.\(^4\) The added mass is a large effect when the surrounding medium is dense and the moving object is not, such as for a ship or air bubble in water.\(^5\)

For a symmetric object the added mass is proportional to the mass of the air displaced:

\[
m_a = \alpha m_{air}, \tag{2}
\]

where \( \alpha \) is a constant that depends on the shape of the object. For an ideal fluid, \( \alpha \) can be calculated from fluid dynamics\(^6\) (see appendix) and is given by

\[
\alpha = \frac{1}{2} \tag{3}
\]

for a sphere. Similarly, for a long, horizontal cylinder, \( \alpha = 1 \). Comparing the added mass and the buoyant force terms in Eq. (1), we see that both are proportional to the mass of the air displaced and thus the two effects are of roughly comparable size for an accelerating projectile.

Here we focus on observing the added mass of various spherical projectiles. Our experiment consists of throwing a ball up in the air and observing its motion with a PASCO ultrasonic motion sensor (see Fig. 1). Care is taken that the throw is vertical, the ball is not rotating, and that the ball is always far from the ceiling or any walls. The position-versus-time data near the top of the motion are fit to a parabola and the average acceleration is extracted. This average acceleration is then used in our calculations. This data analysis procedure minimizes the effects of the drag force in two ways: (1) the drag force has the smallest magnitude here because the speed is small, and (2) by averaging over the up and down motion, the drag force, which points in different directions for the up and down motion, approximately cancels out from the average acceleration.\(^7\) Thus we shall neglect the drag force in all further discussions.

To get a measure of the mass, the balls were all “weighed” on an electronic balance. There the normal force of the balance equals the weight of the object minus the buoyant force. Since the scales are calibrated with a known mass, while the acceleration of gravity \( g \) varies with location, the result of this “weighing” is a mass measurement. We shall denote this mass measurement as \( m_0 \), the mass of the objects in air when the ac-
gravity $g$. Rewriting Eq. (1) in terms of $m_0$, neglecting the drag force, and solving for $g$, we have

$$g = a \left[ 1 + (1 + \alpha) \frac{m_{\text{air}}}{m_0} \right].$$

(5)

Figure 2 shows the measured acceleration $a$ for different balls. Also shown in Fig. 2 is the corresponding gravitational acceleration constant $g$ calculated using Eq. (5) and assuming the value of $\alpha$ given in Eq. (3). We see that, for low-density projectiles like beach balls, the measured acceleration $a$ can be less than half the acceleration constant $g$. However, after correcting for the buoyant force and the added mass, the values for $g$ show very good agreement with each other and with the local Anchorage value of $9.818 \pm 0.002 \text{ m/s}^2$.

Now we shall instead assume the local value for $g$ and extract a value for $a$. Solving Eq. (5) for $a$, the added mass coefficient,

$$\alpha = \left( \frac{g}{a} - 1 \right) \frac{m_0}{m_{\text{air}}} - 1.$$

(6)

This formula involves differences in measured quantities, so the effects of small errors are enhanced. This is seen in the table where the relative errors of the $\alpha$’s are much larger than those of the $g$’s. The average value of $\alpha$ from all the balls is $0.50 \pm 0.01$. This agrees quite well with the value given in Eq. (3) for an ideal fluid.

In general, this makes an excellent lab activity. The students can observe fluid mechanical effects without getting the classroom all wet, and they get to play with balls, which is always fun. The hardest part of the lab is typically calculating the mass of the air displaced, $m_{\text{air}}$. Determining the volume of a ball is difficult to do accurately, and also the density of the air fluctuates a few percent each day, from changes in the pressure and humidity.

Table I. Data from different balls.

<table>
<thead>
<tr>
<th>Diameter (m)</th>
<th>$m_{\text{air}}/m_0$</th>
<th>acceleration (m/s$^2$)</th>
<th>$g$ (m/s$^2$)</th>
<th>$\alpha$ (for $g = 9.82$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90 ± 0.01</td>
<td>0.96 ± 0.03</td>
<td>3.934 ± 0.001</td>
<td>9.6 ± 0.2</td>
<td>0.56 ± 0.05</td>
</tr>
<tr>
<td>0.656 ± 0.001</td>
<td>0.701 ± 0.007</td>
<td>4.771 ± 0.001</td>
<td>9.79 ± 0.05</td>
<td>0.51 ± 0.02</td>
</tr>
<tr>
<td>0.425 ± 0.004</td>
<td>0.42 ± 0.01</td>
<td>6.03 ± 0.01</td>
<td>9.8 ± 0.1</td>
<td>0.50 ± 0.05</td>
</tr>
<tr>
<td>0.361 ± 0.002</td>
<td>0.357 ± 0.007</td>
<td>6.46 ± 0.01</td>
<td>9.92 ± 0.08</td>
<td>0.46 ± 0.03</td>
</tr>
<tr>
<td>0.286 ± 0.001</td>
<td>0.275 ± 0.006</td>
<td>6.898 ± 0.009</td>
<td>9.75 ± 0.07</td>
<td>0.54 ± 0.04</td>
</tr>
<tr>
<td>0.224 ± 0.002</td>
<td>0.223 ± 0.009</td>
<td>7.269 ± 0.008</td>
<td>9.7 ± 0.1</td>
<td>0.58 ± 0.07</td>
</tr>
<tr>
<td>0.4242 ± 0.0005</td>
<td>0.170 ± 0.002</td>
<td>7.844 ± 0.005</td>
<td>9.84 ± 0.02</td>
<td>0.48 ± 0.02</td>
</tr>
<tr>
<td>0.2128 ± 0.0006</td>
<td>0.0516 ± 0.0008</td>
<td>9.140 ± 0.008</td>
<td>9.85 ± 0.02</td>
<td>0.44 ± 0.05</td>
</tr>
<tr>
<td>Weighted averages</td>
<td></td>
<td>9.83 ± 0.01</td>
<td></td>
<td>0.50 ± 0.01</td>
</tr>
</tbody>
</table>

Fig. 2. Plot of measured acceleration $a$ (blue circles) and the corresponding calculated gravitational acceleration $g$ (red squares) vs the mass ratio $m_{\text{air}}/m_0$. The horizontal line is the local value of $g$, 9.82 m/s$^2$.  

We shall use this directly measured quantity in all of our calculations.

We have taken data using a variety of balls and our measurements are given in Table I. Most of the balls were inflatable beach balls, as shown in Fig. 1, but the bottom two entries in the table are for light, inexpensive plastic balls purchased at the local toy store. The inflatable balls were often slightly ellipsoidal and this is reflected in the generally larger uncertainties in their diameters.

First let us analyze our data to extract the acceleration of gravity and the velocity are zero [see Eq. (1)].

$$m_0 = [m - m_{\text{air}}].$$

(4)

The measured acceleration and the velocity are zero [see Eq. (1)].
Appendix: Calculating the added mass

The equations describing fluid flow follow from Newton’s second law and mass conservation. For an ideal fluid (incompressible, no rotations and no viscosity), the fluid velocity, \( \mathbf{u} \), may be written as the gradient of a scalar function, \( \Phi \).

\[
\mathbf{u} = \nabla \Phi,
\]

where the scalar function satisfies Laplace’s equation

\[
\nabla^2 \Phi = 0.
\]

For a sphere of radius \( R \) moving with speed \( U \) along the \( z \)-axis through an infinite fluid, the boundary conditions are that fluid does not flow through the sphere’s surface, \( u_z(r = R) = U \), and that the fluid velocity vanishes far from the sphere. Then the solution for the fluid velocity field is

\[
\mathbf{u} = \frac{1}{2} \left[ 3 \hat{r} (\hat{r} \cdot \mathbf{U}) - \mathbf{U} \right] \frac{R^3}{r},
\]

where \( r \) is the radial distance from the center of the sphere. This form should be familiar from electrostatics; it has the same geometrical dependence as the electric field far from a dipole. The kinetic energy in the fluid is found by integrating (\( r > R \)).

\[
E = \int dV \frac{1}{2} \rho \mathbf{u}^2 = \frac{1}{2} \rho \frac{2 \pi R^4}{3} U^2,
\]

where \( \rho \) is the fluid density. The term in parenthesis is the effective mass of the fluid from the motion of the sphere and is equal to half the mass of the displaced fluid. This is the added mass as given in Eqs. (2) and (3).

It is interesting to note that the pressure can be obtained from Eq. (9) by switching to a reference frame where the sphere is at rest and then using Bernoulli’s equation. The pressure on the surface of the sphere from the motion of the air is

\[
P(\theta) = P_\infty + \frac{1}{2} \rho U^2 \left( 1 - \frac{9}{4} \sin^2 \theta \right),
\]

where \( P_\infty \) is the pressure at infinity. The pressure is symmetric around the sphere, thus there is no net force on the sphere. This result is known as d’Alembert’s paradox. To get a drag force requires dissipation of energy—viscosity.

References


5. Using Eqs. (1), (2), and (3), with \( m_{\text{water}} \) in place of \( m_{\text{air}} \) and \( m_{\text{water}} >> m \), we see that the initial acceleration of a free, spherical bubble is \(-2g\) (upward).


8. This value comes from our observations of the motion of dense objects (pendulums and projectiles) and agrees with the value given by the International Gravity Formula.

9. We have not included the gravitational term of Bernoulli’s equation in Eq. (11), which does give a net force on the sphere, the buoyant force.

John Messer graduated from Palmer High School in Alaska and then attended the U.S. Air Force Academy and later the University of Mississippi, where he earned his Master’s degree in physics. He then taught introductory physics at Northeast Mississippi Community College for 10 years before moving back to Alaska.

Jim Pantaleone got his PhD in theoretical physics from Cornell University in 1985. He was a post-doc for many years before arriving at the University of Alaska Anchorage, where he is now a professor. He has published many papers on the effective mass of neutrinos in matter.