MATHEMATICAL ASSOCIATION OF AMERICA

Solutions Pamphlet

American Mathematics Competitions

62nd Annual

AMC 12 A

American Mathematics Contest 12 A

Tuesday, February 8, 2011

This Pamphlet gives at least one solution for each problem on this year's contest and shows that all problems can be solved without the use of a calculator. When more than one solution is provided, this is done to illustrate a significant contrast in methods, e.g., algebraic *vs* geometric, computational *vs* conceptual, elementary *vs* advanced. These solutions are by no means the only ones possible, nor are they superior to others the reader may devise.

We hope that teachers will inform their students about these solutions, both as illustrations of the kinds of ingenuity needed to solve nonroutine problems and as examples of good mathematical exposition. *However, the publication, reproduction or communication of the problems or solutions of the AMC 12 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules.*

Correspondence about the problems/solutions for this AMC 12 and orders for any publications should be addressed to:

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The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

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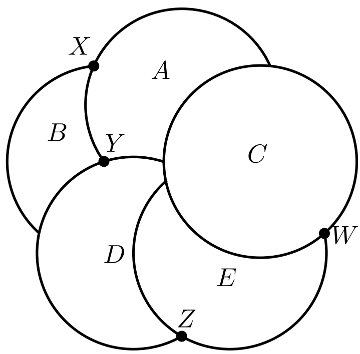
1. **Answer (D):** The text messages cost $\$0.05 \cdot 100 = \$5.00$, and the 30 minutes of excess chatting cost $\$0.10 \cdot 30 = \$3.00$. Therefore the total bill came to $\$5 + \$3 + \$20 = \$28$.

2. **Answer (E):** The circumference of coin B in the figure does not continue past the point X where it intersects the circumference of coin A. Thus coin A is above coin B. Similarly, the points Y, Z, and W in the figure show that coin D is above coin A, coin E is above coin D, and coin C is above coin E, respectively. Thus the order of the coins from top to bottom is (C, E, D, A, B).



3. **Answer (E):** Because $14 \cdot 35 = 490 < 500$ and $15 \cdot 35 = 525 \geq 500$, the minimum number of bottles that she needs to buy is 15.

4. **Answer (C):** Let N equal the number of fifth graders. Then there are $2N$ fourth graders and $4N$ third graders. The total number of minutes run per day by the students is $4N \cdot 12 + 2N \cdot 15 + N \cdot 10 = 88N$. There are a total of $4N + 2N + N = 7N$ students, so the average number of minutes run by the students per day is $\frac{88N}{7N} = \frac{88}{7}$.

5. **Answer (C):** Because 75% of the birds were not swans and 30% of the birds were geese, it follows that $\frac{30}{75} \cdot 100\% = 40\%$ of the birds that were not swans were geese.

6. **Answer (A):** Let x, y, and z be the number of successful three-point shots, two-point shots, and free throws, respectively. Then the given conditions imply

$$3x + 2y + z = 61,$$

$$2y = 3x, \text{ and}$$
$$y + 1 = z.$$

Solving results in $x = 8$, $y = 12$, and $z = 13$. Hence the team made 13 free throws.

7. **Answer (B):** Let C be the cost of a pencil in cents, N be the number of pencils each student bought, and S be the number of students who bought pencils. Then $C \cdot N \cdot S = 1771 = 7 \cdot 11 \cdot 23$, and $C > N > 1$. Because a majority of the students bought pencils, $30 \geq S > \frac{30}{2} = 15$. Therefore $S = 23, N = 7$, and $C = 11$.

8. **Answer (C):** Note that for any four consecutive terms, the first and last terms must be equal. For example, consider B, C, D, and E; because

$$B + C + D = 30 = C + D + E\,,$$

we must have $B = E$. Hence $A = D = G$, and $C = F = 5$. The required sum $A + H = G + (30 - G - F) = 30 - 5 = 25$.

OR

Note that

$$\begin{aligned} A + C + H &= (A + B + C) - (B + C + D) + (C + D + E) \\ &\quad - (E + F + G) + (F + G + H) \\ &= 3 \cdot 30 - 2 \cdot 30 = 30. \end{aligned}$$

Hence $A + H = 30 - C = 25$.

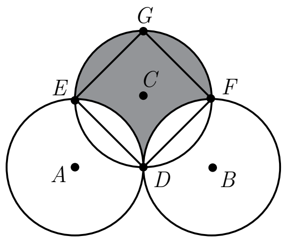
9. **Answer (B):** Each of the 18 twins shook hands with 16 twins and 9 triplets, giving a total of $18 \cdot 25$ handshakes. Similarly, each of the 18 triplets shook hands with 15 triplets and 9 twins, giving a total of $18 \cdot 24$ handshakes. This tally counts every handshake twice, so the number of handshakes is $\frac{1}{2}(18 \cdot 25 + 18 \cdot 24) = 9 \cdot 49 = 441$.

10. **Answer (B):** Let d be the sum of the numbers rolled. The conditions are satisfied if and only if $\pi\left(\frac{d}{2}\right)^2 < \pi d$, that is, $d < 4$. Of the 36 equally likely outcomes for the roll of the two dice, one has a sum of 2 and two have sums of 3. Thus the desired probability is $\frac{1+2}{36} = \frac{1}{12}$.

11. **Answer (C):** Let D be the midpoint of $\overline{AB}$, and let circle C intersect circles A and B at E and F, respectively, distinct from D. The shaded portion of unit square $ADCE$ has area $1-\frac{\pi}{4}$, as does the shaded portion of unit square $BDCF$. The portion of the shaded region which is outside these squares is a semicircle of radius 1 and has area $\frac{\pi}{2}$. The total shaded area is $2\left(1-\frac{\pi}{4}\right)+\frac{\pi}{2}=2$.

OR

Let D, E, and F be defined as in the first solution, and let G be diametrically opposite D on circle C. The shaded area is equal to the area of square $DFGE$, which has diagonal length 2. Its side length is $\sqrt{2}$, and its area is $\left(\sqrt{2}\right)^2=2$.



12. **Answer (D):** Assume the power boat and raft met at point O on the river. Let x be the speed of the boat and y be the speed of the raft and the river current. Then $x+y$ is the speed of the power boat downstream and $x-y$ is the speed of the power boat upstream. Let the distance AB between the docks be S, so that $AO = 9y$ and $OB = S - 9y$. Then because time is equal to distance divided by rate,

$$\frac{S}{x+y}+\frac{S-9y}{x-y}=9\,.$$

Rearrange to find that $S=\frac{9}{2}(x+y)$. Then the time it took the power boat to go from A to B is

$$\frac{S}{x+y}=\frac{9(x+y)}{2(x+y)}=4.5\,.$$

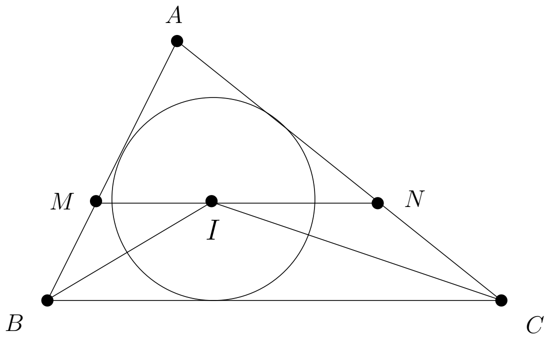
OR

In the reference frame of the raft, the boat simply went away, turned around, and came back, all at the same speed. Because the trip took 9 hours, the boat must have turned around after 4.5 hours.

13. **Answer (B):** Let I be the incenter of $\triangle ABC$. Because I is the intersection of the angle bisectors of the triangle and $\overline{MN}$ is parallel to $\overline{BC}$, it follows that $\angle IBM = \angle CBI = \angle MIB$ and $\angle NCI = \angle ICB = \angle CIN$. Hence $\triangle BMI$ and

$\triangle CNI$ are isosceles with $MB = MI$ and $CN = IN$. Thus the perimeter of $\triangle AMN$ is

$$\begin{aligned} AM + MN + NA &= AM + MI + IN + NA \\ &= AM + MB + CN + NA \\ &= AB + AC = 12 + 18 = 30. \end{aligned}$$



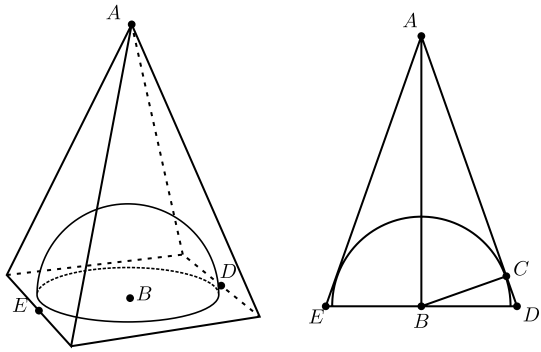
14. **Answer (E):** The point (a, b) is above the parabola if and only if $b > a^3 - ab$. Because a is positive, this is equivalent to $b > \frac{a^3}{a+1}$. If $a = 1$, then b can be any digit from 1 to 9 inclusive. If $a = 2$, then b can be any digit between 3 and 9 inclusive. If $a = 3$, then b can be any digit between 7 and 9 inclusive. If $a > 3$, there is no b that satisfies $b > \frac{a^3}{a+1}$. Therefore there are $9 + 7 + 3 = 19$ pairs satisfying the condition, out of a total of $9 \cdot 9 = 81$ pairs. The requested probability is $\frac{19}{81}$.

15. **Answer (A):** A cross section of the figure is shown, where A is the apex, B is the center of the base, D and E are the midpoints of opposite sides of the base, and the hemisphere meets $\overline{AD}$ at C.

    Right triangle ABC has $AB = 6$ and $BC = 2$, so $AC = 4\sqrt{2}$. Because $\triangle ABC \sim \triangle ADB$, it follows that

$$BD = \frac{BC \cdot AB}{AC} = \frac{2 \cdot 6}{4\sqrt{2}} = \frac{3\sqrt{2}}{2}.$$

    Hence the length of the base edge is $DE = 2 \cdot BD = 3\sqrt{2}$.



16. **Answer (C):** If five distinct colors are used, then there are $\binom{6}{5} = 6$ different color choices possible. They may be arranged in $5! = 120$ ways on the pentagon, resulting in $120 \cdot 6 = 720$ colorings.

    If four distinct colors are used, then there is one duplicated color, so there are $\binom{6}{4}\binom{4}{1} = 60$ different color choices possible. The duplicated color must appear on neighboring vertices. There are 5 neighbor choices and $3! = 6$ ways to color the remaining three vertices, resulting in a total of $60 \cdot 5 \cdot 6 = 1800$ colorings.

    If three distinct colors are used, then there must be two duplicated colors, so there are $\binom{6}{3}\binom{3}{2} = 60$ different color choices possible. The non-duplicated color may appear in 5 locations. As before, a duplicated color must appear on neighboring vertices, so there are 2 ways left to color the remaining vertices. In this case there are $60 \cdot 5 \cdot 2 = 600$ colorings possible.

    There are no colorings with two or fewer colors. The total number of colorings is $720 + 1800 + 600 = 3120$.

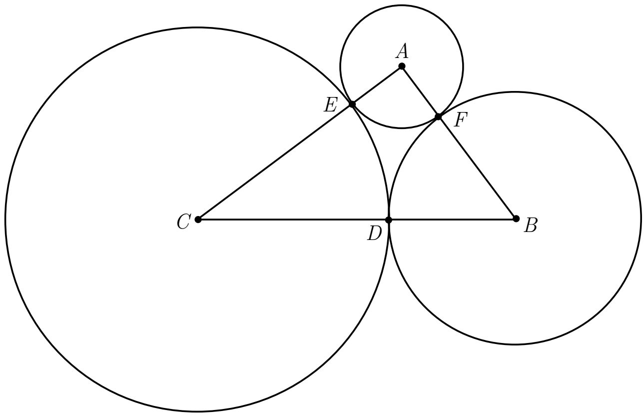
17. **Answer (D):** Let A, B, and C be the centers of the circles with radii 1, 2, and 3, respectively. Let D, E, and F be the points of tangency, where D is on the circles B and C, E is on the circles A and C, and F is on the circles A and B. Because $AB = AF + FB = 1 + 2 = 3$, $BC = BD + DC = 2 + 3 = 5$, and $CA = CE + EA = 3 + 1 = 4$, it follows that $\triangle ABC$ is a 3-4-5 right triangle. Therefore

$$[ABC] = \frac{1}{2}AB \cdot AC = 6, \quad [AEF] = \frac{1}{2}AE \cdot AF = \frac{1}{2},$$
$$[BFD] = \frac{1}{2}BD \cdot BF \cdot \sin(\angle FBD) = \frac{1}{2} \cdot 2 \cdot 2 \cdot \frac{4}{5} = \frac{8}{5}, \text{ and}$$
$$[CDE] = \frac{1}{2}CD \cdot CE \cdot \sin(\angle DCE) = \frac{1}{2} \cdot 3 \cdot 3 \cdot \frac{3}{5} = \frac{27}{10}.$$

Hence

$$[DEF] = [ABC] - [AEF] - [BFD] - [CDE]$$

$$= 6 - \frac{1}{2} - \frac{8}{5} - \frac{27}{10} = \frac{6}{5}.$$



18. **Answer (D):** The graph of the equation $|x+y|+|x-y| = 2$ is a square formed by the lines $x = \pm 1$ and $y = \pm 1$. For $c > -9$, the equation $c = x^2 - 6x + y^2 = (x-3)^2 + y^2 - 9$ is the equation of a circle with center $(3, 0)$ and radius $\sqrt{c+9}$. Among all such circles that intersect the square, the largest one contains the points $(-1, \pm 1)$ and has radius $\sqrt{4^2 + 1^2} = \sqrt{17}$. It follows that the maximum value of c is $17 - 9 = 8$.

19. **Answer (C):** The given conditions imply that $N \geq 19$ and $1+\lfloor \log_2(N-1) \rfloor = \log_2(N+19)$. Because $1 + \lfloor \log_2(N-1) \rfloor$ is a positive integer, so is $\log_2(N+19)$; thus $2^k = N + 19 \geq 38$ for some integer k. It follows that $k \geq 6$ and the two smallest values of N are $2^6 - 19 = 45$ and $2^7 - 19 = 109$, whose sum is 154.

    Note: This formula for the number of elite status players provides a method used to determine the number of first-round byes in a single-elimination tournament.

20. **Answer (C):** Note that $f(1) = a+b+c = 0$, so $f(7) = 49a+7b+c = 48a+6b = 6(8a + b)$. Thus $f(7)$ is an integer multiple of 6 strictly between 50 and 60, so $f(7) = 54$ and $8a+b = 9$. Similarly, $f(8) = 64a+8b+c = 63a+7b = 7(9a+b)$. Thus $f(8)$ is an integer multiple of 7 strictly between 70 and 80, so $f(8) = 77$ and $9a + b = 11$. It follows that $a = 2$, $b = -7$, and $c = 5$. Therefore $f(100) = 2 \cdot 100^2 - 7 \cdot 100 + 5 = 19{,}305$, and thus $k = 3$.

21. **Answer (A):** Because $f_2(x) = \sqrt{1 - \sqrt{4 - x}}$, $f_2(x)$ is defined if and only if $0 \leq \sqrt{4 - x} \leq 1$, so the domain of f_2 is the interval $[3, 4]$. Similarly, the

domain of f_3 is the solution set of the inequality $3 \le \sqrt{9-x} \le 4$, which is the interval $[-7, 0]$, and the domain of f_4 is the solution set of the inequality $-7 \le \sqrt{16-x} \le 0$, which is $\{16\}$. The domain of f_5 is the solution set of the equation $\sqrt{25-x} = 16$, which is $\{-231\}$, and because the equation $\sqrt{36-x} = -231$ has no real solutions, the domain of f_6 is empty. Therefore $N + c = 5 + (-231) = -226$.

22. **Answer (C):** Assume without loss of generality that R is bounded by the square with vertices $A = (0,0)$, $B = (1,0)$, $C = (1,1)$, and $D = (0,1)$, and let $X = (x,y)$ be n-ray partitional. Because the n rays partition R into triangles, they must include the rays from X to A, B, C, and D. Let the number of rays intersecting the interiors of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ be n_1, n_2, n_3, and n_4, respectively. Because $\triangle ABX \cup \triangle CDX$ has the same area as $\triangle BCX \cup \triangle DAX$, it follows that $n_1 + n_3 = n_2 + n_4 = \frac{n}{2} - 2$, so n is even. Furthermore, the $n_1 + 1$ triangles with one side on $\overline{AB}$ have equal area, so each has area $\frac{1}{2} \cdot \frac{1}{n_1+1} \cdot y$. Similarly, the triangles with sides on $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ have areas $\frac{1}{2} \cdot \frac{1}{n_2+1} \cdot (1-x)$, $\frac{1}{2} \cdot \frac{1}{n_3+1} \cdot (1-y)$, and $\frac{1}{2} \cdot \frac{1}{n_4+1} \cdot x$, respectively. Setting these expressions equal to each other gives

$$x = \frac{n_4+1}{n_2+n_4+2} = \frac{2(n_4+1)}{n} \text{ and } y = \frac{n_1+1}{n_1+n_3+2} = \frac{2(n_1+1)}{n}.$$

Thus an n-ray partitional point must have the form $X = (\frac{2a}{n}, \frac{2b}{n})$ with $1 \le a < \frac{n}{2}$ and $1 \le b < \frac{n}{2}$. Conversely, if X has this form, R is partitioned into n triangles of equal area by the rays from X that partition $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ into b, $\frac{n}{2} - a$, $\frac{n}{2} - b$, and a congruent segments, respectively.

Assume X is 100-ray partitional. If X is also 60-ray partitional, then $X = (\frac{a}{50}, \frac{b}{50}) = (\frac{c}{30}, \frac{d}{30})$ for some integers $1 \le a, b \le 49$ and $1 \le c, d \le 29$. Thus $3a = 5c$ and $3b = 5d$; that is, both a and b are multiples of 5. Conversely, if a and b are multiples of 5, then

$$X = \left(\frac{a}{50}, \frac{b}{50}\right) = \left(\frac{\frac{3a}{5}}{30}, \frac{\frac{3b}{5}}{30}\right)$$

is 60-ray partitional. Because there are exactly 9 multiples of 5 between 1 and 49, the required number of points X is equal to $49^2 - 9^2 = 40 \cdot 58 = 2320$.

23. **Answer (C):** Note that

$$g(z) = \frac{\frac{z+a}{z+b} + a}{\frac{z+a}{z+b} + b} = \frac{(1+a)z + a(1+b)}{(1+b)z + (a+b^2)} = \frac{Az+B}{Cz+D},$$

where $A = 1+a$, $B = a(1+b)$, $C = 1+b$, and $D = a+b^2$. Then

$$g(g(z)) = \frac{Ag(z)+B}{Cg(z)+D}.$$

Setting $g(g(z)) = z$ and solving for $g(z)$ gives

$$g(z) = \frac{-Dz + B}{Cz - A}.$$

Equating the two expressions for $g(z)$ gives

$$(Az + B)(Cz - A) = (-Dz + B)(Cz + D),$$

that is,

$$(A + D)(z^2C + z(D - A) - B) = 0.$$

Therefore either $B = C = 0$ (and $A = D$) or $A + D = 0$. In the former case $b = -1$, $f(z) = \frac{z+a}{z-1}$, and $g(z) = \frac{(1+a)z}{1+a} = z$, as required, unless $a = -1$. (Note that $a = -1$ in this case would imply $f(z) = 1$, which contradicts $g(g(z)) = z$.) In the latter case $1 + 2a + b^2 = 0$, so $|b|^2 = |2a + 1|$. Because $|a| = 1$, the triangle inequality yields

$$1 = |2|a| - 1| \le |2a + 1| \le 2|a| + 1 = 3,$$

so $1 \le |b| \le \sqrt{3}$. The minimum $|b| = 1$ is attained when $a = -1$ and $b = 1$ (or as above, when $b = -1$). The maximum $|b| = \sqrt{3}$ is attained when $a = 1$ and $b = \pm\sqrt{3}i$. The required difference is $\sqrt{3} - 1$.

Note: The conditions imply that a lies on the unit circle in the complex plane, so $2a+1$ lies on a circle of radius 2 centered at 1. The steps above are reversible, so if $b^2 = -1 - 2a$, then $g(g(z)) = z$ (unless $a = b = -1$). Therefore b^2 can be anywhere on the circle of radius 2 centered at -1, and $|b|$ can take on any value between 1 and $\sqrt{3}$.

24. **Answer (C):** Because $AB + CD = 21 = BC + DA$, it follows that $ABCD$ always has an inscribed circle tangent to its four sides. Let r be the radius of the inscribed circle. Note that $[ABCD] = \frac{1}{2}r\,(AB + BC + CD + DA) = 21r$. Thus the radius is maximum when the area is maximized. Note that $[ABC] = \frac{1}{2} \cdot 14 \cdot 9 \sin B = 63 \sin B$ and $[ACD] = \frac{1}{2} \cdot 12 \cdot 7 \sin D = 42 \sin D$. On the one hand,

$$\begin{aligned}[ABCD]^2 &= ([ABC] + [ACD])^2 \\ &= 63^2 \sin^2 B + 42^2 \sin^2 D + 2 \cdot 42 \cdot 63 \sin B \sin D.\end{aligned}$$

On the other hand, by the Law of Cosines,

$$AC^2 = 12^2 + 7^2 - 2 \cdot 7 \cdot 12 \cos D = 14^2 + 9^2 - 2 \cdot 9 \cdot 14 \cos B.$$

Thus

$$21^2 = \left(\frac{2 \cdot 26 + 2 \cdot 16}{4}\right)^2 = \left(\frac{14^2 - 12^2 + 9^2 - 7^2}{4}\right)^2 = (63 \cos B - 42 \cos D)^2$$

$$= 63^2\cos^2 B + 42^2\cos^2 D - 2\cdot 42\cdot 63\cos B\cos D.$$

Adding these two identities yields

$$\begin{aligned}[ABCD]^2 + 21^2 &= 63^2 + 42^2 - 2\cdot 42\cdot 63\cos(B+D)\\ &\le 63^2 + 42^2 + 2\cdot 42\cdot 63 = (63+42)^2 = 105^2,\end{aligned}$$

with equality if and only if $B + D = \pi$ (that is $ABCD$ is cyclic). Therefore $[ABCD]^2 \le 105^2 - 21^2 = 21^2(5^2-1) = 42^2\cdot 6$, and the required maximum $r = \frac{1}{21}[ABCD] = 2\sqrt{6}$.

OR

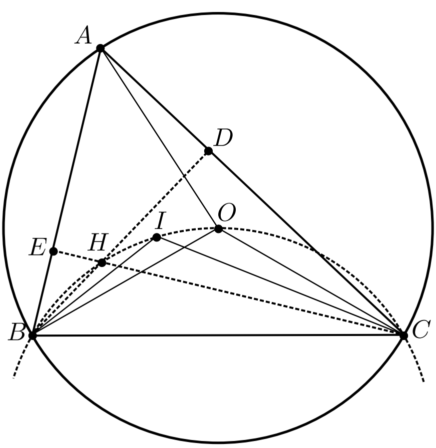
Establish as in the first solution that r is maximized when the area is maximized. Bretschneider's formula, which generalizes Brahmagupta's formula, states that the area of an arbitrary quadrilateral with side lengths a, b, c, and d, is given by

$$\sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\theta},$$

where $s = \frac{1}{2}(a+b+c+d)$ and θ is half the sum of either pair of opposite angles. For a, b, c, and d fixed, the area is maximized when $\cos\theta = 0$. Thus the area is maximized when $\theta = \frac{1}{2}\pi$, that is, when the quadrilateral is cyclic. In this case, the area equals $\sqrt{7\cdot 12\cdot 14\cdot 9} = 42\sqrt{6}$, and the required maximum radius $r = \frac{1}{21}\cdot 42\sqrt{6} = 2\sqrt{6}$.

25. **Answer (D):** By the Inscribed Angle Theorem, $\angle BOC = 2\angle BAC = 120^\circ$. Let D and E be the feet of the altitudes of $\triangle ABC$ from B and C, respectively. Because $\overline{CE}$ and $\overline{BD}$ intersect at H,

$$\begin{aligned}\angle BHC = \angle DHE &= 360^\circ - \angle HEA - \angle ADH - \angle EAD\\ &= 360^\circ - 90^\circ - 90^\circ - 60^\circ = 120^\circ.\end{aligned}$$



Because the lines BI and CI are bisectors of $\angle CBA$ and $\angle ACB$, respectively, it follows that

$$\begin{aligned}\angle BIC &= 180^\circ - \angle ICB - \angle CBI = 180^\circ - \frac{1}{2}\left(\angle ACB + \angle CBA\right)\\ &= 180^\circ - \frac{1}{2}\left(180^\circ - \angle BAC\right) = 120^\circ.\end{aligned}$$

Thus the points B, C, O, I, and H are all on a circle. Further,

$$\begin{aligned}\angle OCI = \angle ACI - \angle ACO &= \frac{1}{2}\angle ACB - \left(90^\circ - \frac{1}{2}\angle COA\right)\\ &= \frac{1}{2}\angle ACB - (90^\circ - \angle CBA)\\ &= \frac{1}{2}\angle ACB - 90^\circ + (120^\circ - \angle ACB) = 30^\circ - \frac{1}{2}\angle ACB,\end{aligned}$$

and $\angle ICH = \angle ACH - \angle ACI = (90^\circ - \angle EAC) - \frac{1}{2}\angle ACB = 30^\circ - \frac{1}{2}\angle ACB$. Thus $OI = IH$. Because $[BCOIH] = [BCO] + [BOIH]$ and BCO is an isosceles triangle with $BC = 1$ and $OB = OC = \frac{1}{\sqrt{3}}$, it is sufficient to maximize the area of quadrilateral $BOIH$. If P_1, P_2 are two points in an arc of circle BO with $BP_1 < BP_2$, then the maximum area of BOP_1P_2 occurs when $BP_1 = P_1P_2 = P_2O$. Indeed, if $BP_1 \neq P_1P_2$, then replacing P_1 by the point P_1' located halfway in the arc of circle BP_2 yields a triangle $BP_1'P_2$ with larger area than $\triangle BP_1P_2$, and the area of $\triangle BOP_2$ remains the same. Similarly, if $P_1P_2 \neq P_2O$, then replacing P_2 by the midpoint P_2' of the arc P_1O causes the area of $\triangle P_1P_2'O$ to increase and the area of $\triangle BP_1O$ to remain the same.

Therefore the maximum is achieved when $OI = IH = HB$, that is, when $\angle OCI = \angle ICH = \angle HCB = \frac{1}{3}\angle OCB = 10^\circ$. Thus $30^\circ - \frac{1}{2}\angle ACB = 10^\circ$, so $\angle ACB = 40^\circ$ and $\angle CBA = 80^\circ$.

The problems and solutions in this contest were proposed by Bernardo Abrego, Betsy Bennett, Steve Dunbar, Michelle Ghrist, Peter Gilchrist, Jerrold Grossman, Dan Kennedy, Joe Kennedy, David Torney, David Wells, LeRoy Wenstrom, and Ron Yannone.

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